Grade 7/8 Math Circles

November 6/7/8/9

Geometric Sequences Solutions

Exercise Solutions

We will provide solutions to select exercises from the handout.

2. $3^4 = 81$, $2^6 = 64$, $\left(\frac{1}{2}\right) = \frac{1}{8}$.
3. $t_2 = 4$, $t_5 = 32$, $t_8 = 256$.
4. (a) Geometric, $r = 2$.
   (b) Not geometric.
   (c) Not geometric.
   (d) Geometric, $r = 3$
   (e) Geometric, $r = \frac{1}{2}$
6. $a = 2$, $r = 3$, $n = 4$. Of course, this series would be easy to add without the formula, but it's worth seeing.

$$2 + 6 + 18 + 54 = 2 \times (1 - 3^4) \div (1 - 3) = 2 \times (1 - 81) \div (-2) = 2 \times (-80) \div (-2) = 80$$

Problem Set Solutions

1. We will denote the common ratio of a geometric sequence r, and the common difference of an arithmetic sequence d.
   (a) Arithmetic, $d = 5$
   (b) Geometric, $r = 4$
   (c) Arithmetic, $d = -3$
   (d) Neither
   (e) Geometric, $r = 1$
   (f) Neither
2. (a) 75
   (b) 341

(c) -15

(d) 13

(e) 10

(f) 17

3. $\{5, -10, 20, -40, 80\}$
   $\{5, 3, 1, -1, -3\}$

4. Since we know that the sequence is geometric, there is a common ratio. Notice that to go from 6 to 12, we multiply by 2, and to go from 12 to 24, we again multiply by 2, so we can see that the common ratio is $r = 2$. Hence $a = \frac{3}{2} \times 2 = 3$ and $b = 24 \times 2 = 48$.

5. (a) $a = \frac{4}{2}$, $r = \frac{3}{4}$, $n = 4$

$$\frac{4}{2} \times \left(1 - \left(\frac{3}{4}\right)^4\right) \div \left(1 - \frac{3}{4}\right) = \frac{4}{2} \times \left(1 - \frac{81}{256}\right) \div \frac{1}{4} = \frac{175}{32}$$

(b) $a = 4$, $r = \frac{1}{2}$, n is arbitrarily large.

$$4 \times \left(1 - \left(\frac{1}{2}\right)^n\right) \div \left(1 - \frac{1}{2}\right) = 4 \times (1 - 0) \div \frac{1}{2} = 8$$

(c) $a = -1$, $r = -3$, $n = 5$

$$(-1) \times \left(1 - (-3)^5\right) \div (1 - (-3)) = (-1) \times (1 - (-243)) \div (4) = -61$$

(d) $a = \frac{1}{3}$, $r = \frac{1}{3}$, n is arbitrarily large.

$$\frac{1}{3} \times \left(1 - \left(\frac{1}{3}\right)^n\right) \div \left(1 - \frac{1}{3}\right) = \frac{1}{3} \times (1 - 0) \div \frac{2}{3} = \frac{1}{2}$$

6. (a) We can write

$$\frac{1}{3} = 0.\dot{3} = 0.3 + 0.03 + 0.003 + 0.0003 + \cdots$$

(b) $a = 0.3$, $r = 0.1$, and n is is arbitrarily large.

(c)

$$0.3 \times (1 - 0.1^n) \div (1 - 0.1) = 0.3 \times (1 - 0) \div (0.9) = \frac{1}{3}$$

as expected.